New Efficient Full Wave Optimization of Microwave Circuits by the Adjoint Network Method

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***Abstract*—For the first time, the adjoint network method, which is well known in the area of circuit theory, is applied directly to the full wave model of microwave circuits. In this manner, the gradient of the objective function, which depends on all geometrical parameters of the microwave structure, is computed by just one full wave analysis, thus reducing dramatically the computational effort required by the optimization.**

I. Introduction

THE design of microwave passive components is based on the following steps:

1) First, an approximate lumped or distributed circuit model is synthesized using conventional circuit theory.

2) The circuit model is converted into its microwave counterpart; i.e., all the parameters that define the physical structure of the component are determined.

3) Because of the approximations involved in the circuit model, which typically neglects a number of effects due to unwanted reactances and higher order mode effects, a final optimization based on full wave models is required in order that the microwave component satisfy the requirements.

The most efficient optimization techniques are based on the knowledge of the gradient of the objective function, i.e., its first derivatives with respect to all geometrical parameters. Because of the complexity of the full wave model of the component, the derivatives must be evaluated numerically, i.e., by finite difference approximation. For a component with M geometrical parameters, the computation of the objective function and its gradient requires $M + 1$ full wave analyses. Since the optimization usually requires several hundreds iterations and the number of geometrical parameters is easily of the order of several tenths, the overall numerical effort for step 3) can easily become exceedingly heavy.

It is the purpose of this letter to introduce the adjoint network method (ANM), already well known in circuit theory, in the full wave CAD of microwave circuits. The ANM allows the computation of the gradient of the objective function with just one analysis of the microwave circuit. Here we consider the case of microwave circuits that are bounded by metal walls. In this case, since the admittance matrix of each constituent subnetwork has a simple analytical expression, a dramatic saving of the computer effort involved by the full wave optimization is achieved.

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II. The Adjoint Network Method

Let us consider a generic microwave circuit consisting of the connection of a number of subnetworks (including line or waveguide lengths). Using the microwave network formalism, we associate voltages and currents to the modal transverse electric and magnetic fields at the ports of each subnetwork. By denoting with v_p, i_p voltages and currents at the external ports (the index c will be used to denote internal (or connected) ports), the terminal description of the network can be written in terms of the admittance matrix

$$\mathbf{I}_p = \hat{\mathbf{Y}}\mathbf{V}_p. \tag{1}$$

Optimization routines based on the gradient of a properly defined objective function require ultimately the knowledge of the gradient of the admittance matrix $\hat{\mathbf{Y}}$ (or any other terminal description, such as the scattering matrix). The latter is not known in an analytical form apart from very simple cases. Therefore it must be computed numerically by finite differences, i.e., by taking a finite variation of each geometrical parameter and computing the corresponding variation in $\hat{\mathbf{Y}}$.

This extremely lengthy and inefficient procedure is avoided by the ANM. The adjoint network is defined according to [1], [2]. In our case, due to reciprocity, it coincides with the microwave circuit itself. By modifying the formulation in [2] from the scattering matrix to the admittance matrix representation, one obtains the derivative of $\hat{\mathbf{Y}}$ with respect to any geometrical parameter $x_m (m = 1, \ldots M)$ in the form

$$\frac{\partial \hat{\mathbf{Y}}_{jk}}{\partial x_m} = \sum_{i=1}^{N_p} (\mathbf{V}_i^T)_j \frac{\partial \mathbf{Y}_i^T}{\partial x_m} (\mathbf{V}_i)_k \tag{2}$$

where

- the $(\mathbf{V}_i)_j$ are the voltages at the ports of the subnetwork i when the latter is excited from the external port j;
- $\mathbf{Y}_i$ is the admittance matrices of the ith subnetwork
- the superscript T denotes transposition.

Now, considering microwave circuits bounded by metal walls, the admittance matrix can be evaluated analytically with no matrix inversion [3]–[6] so that the derivatives in the r.h.s. of (2) can be calculated in analytical form. The voltages at the external ports are known by hypothesis (they are either zero or one depending on which port is excited), while those at the N_c connected ports can be computed as follows.

First, from the admittance matrix descriptions of the individual subnetworks, by suitably grouping currents and voltages,

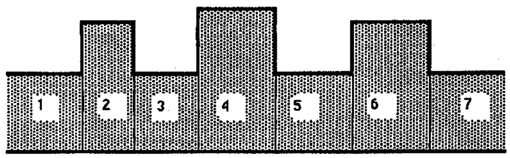

Fig. 1(a). Schematic of a waveguide bandpass filter realized by the cascade of waveguide sections (E-plane steps).

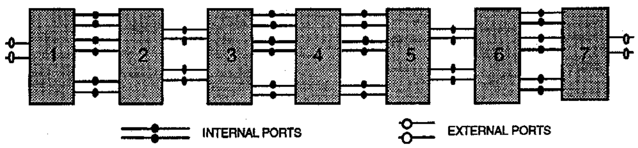

Fig. 1(b). Generalized equivalent network of the structure of Fig. 1(a).

we may write

$$\begin{pmatrix} \mathbf{I}_p \\ \mathbf{I}_c \end{pmatrix} = \begin{pmatrix} \mathbf{y}_{pp} & \mathbf{y}_{pc} \\ \mathbf{y}_{cp} & \mathbf{y}_{cc} \end{pmatrix} \begin{pmatrix} \mathbf{V}_p \\ \mathbf{V}_c \end{pmatrix} \tag{3}$$

where the index c refers to the connected ports.

We can always assume that ports are connected in pairs. A connection matrix can then be introduced to express the topological relations among the connected ports, i.e., the equality of the voltages and, apart from the sign, of the currents at the ports connecting two subnetworks. In this manner the currents at the connected ports can be eliminated from (3) to obtain the following matrix relation:

$$\begin{pmatrix} \mathbf{I}_p \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} \mathbf{y}_{pp} & \mathbf{y}'_{pc} \\ \mathbf{y}'_{cp} & \mathbf{y}'_{cc} \end{pmatrix} \begin{pmatrix} \mathbf{V}_p \\ \mathbf{V}_c \end{pmatrix} \tag{4}$$

where $\mathbf{y}_{pp}$ is the same as in (3) while the other submatrices are expressed in terms of admittance submatrices (3) and the connection matrix.

From the second row of the above matrix equation we can compute the voltages at the connected ports $\mathbf{V}_c$ in terms of the voltages at the external ports $\mathbf{V}_p$. It is important to observe that only one set of equations with coefficient matrix $\mathbf{y}'_{cc}$ has to be solved, thus only one analysis of the microwave network is required.

III. Optimization of a Waveguide Filter

As an example, consider a waveguide bandpass filter consisting of the cascade of seven rectangular waveguide sections of different heights. Using the cellular approach [3] the structure is seen as the connection of 3 cavities (2, 4, 6) and 4 waveguide lengths (1, 3, 5, 7). Observe that the latter have basically a simple diagonal admittance matrix. Fig. 1(a) shows the schematic of the filter structure, while Fig. 1(b) shows its generalized equivalent network representation. Only two external ports can be considered (thus $N_p = 2$), provided that the reference planes are located far enough from the terminal junctions. Nonetheless, all the necessary higher-order modes excited at the various discontinuties can be taken into account and are represented by the connected ports.

A suitable objective function can be defined in terms of the scattering parameters of the filter

$$F = k_1(s_{11} - s_{11}^{obj})^2 + k_2(s_{21} - s_{21}^{obj})^2 \tag{5}$$

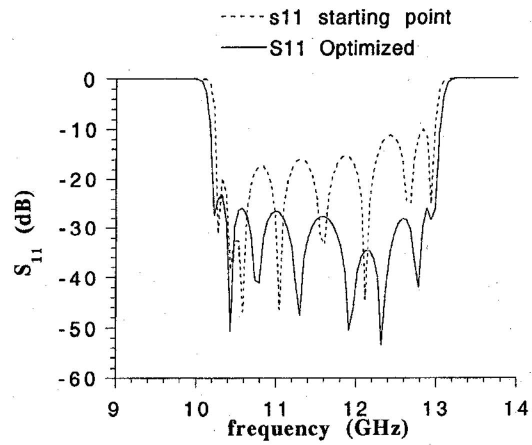

Fig. 2. Reflection coefficient of a filter optimized by the adjoint network method.

where k_1 and k_2 are constant weights to be chosen by the user. To compute the gradient of F, we must evaluate the derivatives of the scattering parameters with respect to all geometric parameters x_m (two for each internal network, thus $m = 1, 2, \ldots 10$). Such derivatives are related to those given by (2) as described in [2]:

$$\frac{\partial \mathbf{S}}{\partial x_m} = -2\sqrt{\mathbf{Y_c}}(\mathbf{Y_c} + \hat{\mathbf{Y}})^{-1}\frac{\partial \hat{\mathbf{Y}}}{\partial x_m}(\mathbf{Y_c} + \hat{\mathbf{Y}})^{-1}\sqrt{\mathbf{Y_c}} \tag{6}$$

where the matrix $\mathbf{Y}_c$ is composed by the characteristic admittances of the feeding lines (1 and 7). Observe that (6) requires the inversion of just a 2×2 matrix ($N_p = 2$)

On this basis, we have designed and optimized a symmetrical bandpass filter consisting of 17 waveguide sections. Because of symmetry, the optimization has been performed by simultaneously changing only 16 geometrical dimensions (heights and lengths of the internal sections). The desired electrical characteristics of the filter have been specified over the band 10.2 GHz to 13 GHz. The objective function has been calculated at 30 frequency points within this band. For comparison, both the conventional FD computation and the adjoint method have been employed. The simulations have been performed on a 486 33-MHz PC. Fig. 2 shows the filter response prior and after optimization(s). Convergence was reached in about the same number of iterations (200) in both cases and the final results are equivalent. However, each iteration required 10 s with the ANM, while 116 s were necessary with FD. An acceleration of 12 times has therefore been obtained.

IV. Conclusion

By directly applying the adjoint network method to the full wave model of a microwave structure, we have demonstrated the possibility of computing analytically the derivatives of the objective function with respect to the geometrical parameters of the structure. In this manner, gradient based optimization routines operating directly on the full wave model can be speeded up dramatically with respect to the conventional finite difference approximation, thus opening new possibilities in the full wave CAD of microwave circuits.

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